Counting Statistics

Radioactive decays follow some curious rules that are a consequence of quantum mechanics. Regardless of when a particular nucleus was created, all nuclei of the same species (cobalt-60 in this experiment) have exactly the same *probability* of decay. We might expect that the longer a nucleus has been around, the more likely it is to decay, but that is not what is observed. Even though the probability that a given nucleus will decay is fixed, there is no way to predict *when* it will decay. In this sense the decay process is completely random. Despite this randomness, a collection of many identical and independent nuclei will exhibit certain predictable behaviors, such as a consistent average decay rate when measured over a long time.

There are still variations in the average count rate when measured over a shorter time, however. Suppose we collect data on the number of decays during a five-second interval. We count decays for five seconds, and then another five, and so forth. If the average number of counts during each interval is n, then we will find that the standard deviation of the collection of measurements is on average $n^{1/2}$. The standard deviation is a measure of how far away, on average, a measurement is from the mean value. A histogram of the measurements of the number of decays detected each interval will show the characteristic distribution known as the *Poisson distribution*.

When the average number of decays each interval is small, such as one or two, then the Poisson distribution is not symmetric. An asymmetric distribution means that the most common value is different from the average value. If the average number of decays in each time interval is larger, such as more than twenty, the shape of the Poisson distribution approaches the shape of the Normal, or Gaussian, distribution. The Normal distribution is sometimes called the *bell-shaped curve*, although there are other distributions that also look like a bell! The Normal distribution is symmetric, with the average value being identical to the most common value.

In this experiment you will collect data from a source that exhibits an essentially constant decay rate. Because the lifetime of the source is so long, the average decay rate will not change during your experiment. The interval-to-interval count rate will vary, however, but in a way consistent with the Poisson distribution.

OBJECTIVES

- Use a radiation counter to determine the distribution of count rates from a nearly constant-rate source.
- Compare the distribution of experimental nuclear counting data to the Poisson distribution.
- Observe the gradual transition of count distribution from Poisson statistics to Gaussian statistics as the average count rate increases.

MATERIALS

TI Graphing Calculator
LabPro or CBL 2
DataRad calculator program
Vernier Radiation Monitor
cobalt-60 1µC gamma source

PRELIMINARY QUESTIONS

1. Connect the Radiation Monitor to a DIG port on your interface, and start the data-collection software if it doesn't start automatically. Place the Radiation Monitor about 10 centimeters from your Co-60 source. When the monitor detects a by-product of a decaying Co-60 nucleus (a gamma ray, in this case) the red LED flashes. Is there a uniform time between flashes or does the time vary? By observing the sequence of flashes, can you predict when the next flash will occur?

2. Now move the source closer to the monitor. Did the average rate of flashes appear to change? If so, how did it change? Is there any more or less uniformity to the time interval between flashes compared to the slower rate?

PROCEDURE

1. Connect the Radiation Monitor to a DIG port of LabPro or CBL 2 if it is not already connected. Use the black link cable to connect the TI graphing calculator to the interface. Firmly press in the cable ends.

2. Turn on the calculator and start the DataRad program. Press CLEAR to reset the program.

3. Prepare the DataRad program for this experiment.
   a. Select SETUP from the main screen.
   b. Select SET INTERVAL from the SETUP MENU.
   c. Select SET INTERVAL from the INTERVAL SETTINGS menu.
   d. Enter **100** as the count time interval in seconds. Always complete number entries with ENTER.
   e. Select OK from the INTERVAL SETTINGS menu.
   f. Select SINGLE INTERVAL from the SETUP MENU.

4. Position the Radiation Monitor next to the Co-60 source so that the rate of flashing of the red LED is maximized. Then, select START from the main screen to start counting.

5. The calculator will begin counting the number of gamma photons that strike the detector during the 100-second count interval. Do not move the detector or the source for the remainder of data collection.

6. After data collection is complete, the number of counts/interval detected will be shown on the calculator screen. Enter this value in your data table. Press ENTER to return to the main screen.

7. To study the variation in count rate distributions, you will need to change the length of one time interval so that the average number of counts is first small (1 or so) and then larger (30 or so). From the count rate measured in the previous step, determine the necessary interval lengths to achieve an average of one count per interval and an average of thirty counts per interval. Round the values up to the next 0.05 s. For example, let's say your average count rate was 530 counts during your 100-second interval. To get about one count per interval with the same source, you would use an interval of $(530/100)^{-1}$ or 0.18 seconds, rounded to 0.20 seconds. For 30 counts, multiply this by 30, getting 6.0 seconds. Enter these rounded values in your data table.

8. Set the counting interval to the value needed to obtain an average count of approximately one.
   a. Select SETUP from the main screen.
   b. Select SET INTERVAL from the SETUP MENU.
   c. Select SET INTERVAL from the INTERVAL SETTINGS screen.
   d. Enter "yy" as the count time interval in seconds, where yy is the count interval you determined for the lower average count rate.
   e. Select OK from the INTERVAL SETTINGS screen.
   f. Select RATE/HISTOGRAM from the SETUP MENU.

9. Select START to begin counting. Observe the count rate values as data are collected. Is there a regular pattern as to the next count rate that appears? Do the values appear to be clustered around a most-common value? Continue collecting data until at least 200 intervals have elapsed. Some time after the 200th interval, press STO▸ to end data collection.

10. DataRad will display a histogram of the measured count rates. Note that the DataRad program reports the count rate in counts per minute, so these values will be large. Initially there may be a small number of bins in the histogram, which may introduce artificial gaps in the graph. To see the histogram with a different number of bins, press ENTER and select YES. Enter a number to see the same data in a histogram with a different number of bins. Print or sketch your histogram.

11. Press ENTER and select NO to see the count statistics. Record the average and standard deviation count rates in your data table.

12. Now set the counting interval to the value needed to obtain an average count of approximately thirty. The larger average count rate will significantly change the shape of the

distribution of count rates. Use the same method as you did earlier to increase the interval setting, and select the rate/histogram mode.

13. Select START to begin counting. Observe the histogram as data are collected. Is there a regular pattern as to the next count rate that appears? Do the values appear to be clustered around a most-common value? Continue collecting data until 200 intervals have elapsed. After the 200th interval, press STO to end data collection.

14. Create your own two-column data table for the histogram data. Label one column “bin max,” or the maximum count value in a histogram bin. Label the other “number,” or the number of times that measurements of that range occurred. Use the ◀ and ▶ keys to read the 10 pairs of values from your histogram, recording them in your data table. You will use this table in Analysis. Print or sketch your histogram.

15. Press ENTER and select NO to see the count statistics. Record the average and standard deviation in your main data table.

16. The standard deviation is a measure of how far away, on average, a typical measurement (of counts during each interval) is from the average of all the measurements. The interval defined by (average ± one standard deviation) contains most of the measurements. From your average and standard deviation values, determine this interval, rounded to the nearest integer. Then, from your histogram data table, mark the first bin that contains counts corresponding to average – one standard deviation, remembering that the bin labels you recorded were the maximum values. Similarly mark the last bin that contains counts corresponding to average + one standard deviation. Using the total of the number of counts in these bins and those in between, determine the fraction of the measurements that fall within the interval (average ± one standard deviation).

DATA TABLE

Counts/interval (100 s interval)	

	Low count rate (~1/interval)	High count rate (~30/interval)
Interval length (s)		
Average rate (cpm)		
Average counts		
Square root (average rate)		
Standard deviation (cpm)		
Standard deviation (counts)		
Fraction within ± std dev		

ANALYSIS

1. Is your first histogram (with the low average count rate) symmetric? How can you tell? Is that shape consistent with the Normal distribution?

2. Is your second histogram (with the high average count rate) approximately symmetric? How can you tell? Is the symmetry of your data distribution consistent with the Normal distribution?

3. The DataRad program reports count rates in cpm, or counts per minute. Knowing the interval length you used, calculate the number of counts actually detected during an average interval. Enter these values in your data table. Similarly, the program reports the standard deviation of the measurements in cpm. Convert this value to the standard deviation in counts detected during each interval. Enter these values in your data table.

4. Calculate the square root of the average count rate for your low and high count rate trials. The square root of the number of counts measured in one interval is an estimate of the standard deviation of a set of measurements, when those measurements follow the Poisson distribution. How does the square-root estimate compare to the actual standard deviation of your set of measurements?

5. Use the comparison in the previous question to answer this question: An experiment yields 900 counts in one interval. Predict the standard deviation of a set of 200 additional measurements made under the same conditions.

6. Is the fraction of the measurements that fall within the interval close to two-thirds? The Normal distribution is symmetric and has two-thirds of its values within one standard deviation of the average. Is the distribution of your data consistent with the Normal distribution? Remember that you used only ten bins in the histogram, and so your fraction includes count rates not in the desired interval.

EXTENSIONS

1. Use additional bins (45) in your histograms to make more careful determinations of the fraction of measurements that fall within one standard deviation of the average count rate. Use these more detailed histograms in all of the investigations below.

2. Consult a statistics or nuclear physics reference book to learn the mathematical form of the Poisson distribution. Plot a Poisson distribution with the same average and standard deviation as your low-count-rate data on the same graph with those data.

3. Consult a statistics or nuclear physics reference book to learn the mathematical form of the Normal distribution. Plot a Normal distribution with the same average and standard deviation as your high-count-rate data on the same graph with those data.

4. Determine the fraction of your measurements falling with two standard deviations of the average for the high-count-rate measurements. The Normal distribution includes 90% of the measurements within two standard deviations of the average.

5. Determine the fraction of your measurements falling with three standard deviations of the average for the high-count-rate measurements. The Normal distribution includes 99% of the measurements within two standard deviations of the average.