# Lifetime Measurement

The *activity* (in decays per second) of some radioactive samples varies in time in a particularly simple way. If the activity ($R$) in decays per second of a sample is proportional to the amount of radioactive material ($R \propto N$, where $N$ is the number of radioactive nuclei), then the activity must decrease in time exponentially:

$$R(t) = R_0 e^{-\lambda t}$$

In this equation λ is the *decay constant*, commonly measured in $s^{-1}$ or $min^{-1}$. $R_0$ is the activity at $t = 0$. The SI unit of activity is the becquerel (Bq), defined as one decay per second.

You will use a source called an isogenerator to produce a sample of radioactive barium. The isogenerator contains cesium-137, which decays to barium-137. The newly made barium nucleus is initially in a long-lived excited state, which eventually decays by emitting a gamma photon. The barium nucleus is then stable, and does not emit further radiation. Using a chemical separation process, the isogenerator allows you to remove a sample of barium from the cesium-barium mixture. Some of the barium you remove will still be in the excited state and will subsequently decay. It is the activity and lifetime of the excited barium you will measure.

While the decay constant λ is a measure of how rapidly a sample of radioactive nuclei will decay, the *half-life* of a radioactive species is also used to indicate the rate at which a sample will decay. A half-life is the time it takes for half of a sample to decay. That is equivalent to the time it takes for the activity to drop by one-half. Note that the half-life (often written as $t_{1/2}$) is not the same as the decay constant λ, but they can be determined from one another.

*Follow all local procedures for handling radioactive materials.* Follow any special use instructions included with your isogenerator.

## OBJECTIVES

- Use a radiation counter to measure the decay constant and half-life of barium-137.
- Determine if the observed time-variation of radiation from a sample of barium-137 is consistent with simple radioactive decay.

## MATERIALS

| | |
|---|---|
| computer | Radiation Monitor |
| Vernier computer interface | Cesium/Barium-137 Isogenerator |
| Logger *Pro* | cut-off paper cup for Barium solution |

## PRELIMINARY QUESTIONS

1. Consider a candy jar, initially filled with 1000 candies. You walk past it once each hour. Since you don't want anyone to notice that you're taking candy, each time you take 10% of the candies remaining in the jar. Sketch a graph of the number of candies for a few hours.

2. How would the graph change if instead of removing 10% of the candies, you removed 20%? Sketch your new graph.

## PROCEDURE

1. Prepare a shallow cup to receive the barium solution. The cup sides should be no more than 1 cm high.

2. Connect the radiation monitor to DIG/SONIC 1 of the computer interface. Turn on the monitor.

3. Prepare the computer for data collection by opening the file "03 Lifetime" from the *Nuclear Radiation w Vernier* folder of Logger *Pro*. One graph is displayed: count rate *vs*. time. The vertical axis is scaled from 0 to 1200 counts/interval. The horizontal axis is time scaled from 0 to 30 minutes.

4. Prepare your isogenerator for use as directed by the manufacturer. Extract the barium solution into the prepared cup. Work quickly between the time of solution extraction and the start of data collection in step 6, for the barium begins to decay immediately.

5. Place the radiation monitor on top of or adjacent to the cup so that the rate of flashing of the red LED is maximized. Take care not to spill the solution.

6. Click ▸ Collect to begin collecting data. Logger *Pro* will begin counting the number of gamma photons that strike the detector during each 30 second count interval. Data collection will continue for 30 minutes. Do not move the detector or the barium cup during data collection.

7. After data collection is complete, the ▸ Collect button will reappear. Set the radiation monitor aside, and dispose of the barium solution and cup as directed by your instructor.

## DATA TABLE

| | |
|---|---|
| Average background counts | |
| fit parameters for Y = A exp ( – C*X ) + B | |
| A | |
| B | |
| C | |
| $\lambda$ (min$^{-1}$) | |
| $t_{1/2}$ (min) | |

## ANALYSIS

1. Inspect your graph. Does the count rate decrease in time? Is the decrease consistent with an activity proportional to the amount of radioactive material remaining?

2. Compare your graph to the graphs you sketched in the Preliminary Questions. How are they different? How are they similar? Why are they similar?

3. The solution you obtained from the isogenerator may contain a small amount of long-lived cesium in addition to the barium. To account for the counts due to any cesium, as well as for counts due to cosmic rays and other background radiation, you can measure the background count rate from your data. By taking data for 30 minutes, the count rate should have gone down to a nearly constant value, aside from normal statistical fluctuations. The counts during

each interval in the last five minutes should be nearly the same as for the 20 to 25 minute interval. If so, you can use the average rate at the end of data collection to measure the counts not due to barium. To do this,

a. Select the data on the graph between 25 and 30 minutes by dragging across the region with your mouse.
b. Click on the statistics button on the toolbar.
c. Read the average counts during the intervals from the floating box, and record the value in your data table. You will use this value to compare with a curve fit parameter.

4. Fit an exponential function to the first fifteen minutes of your data:

   a. Select the first fifteen minutes of points on the graph using the mouse.
   b. Click the Curve Fit button. Select Natural Exponential from the equation list.
   c. Click Try Fit. A best-fit curve will be displayed on the graph. If your data follow the exponential relationship, the curve should closely match the data. When you are satisfied with the fit, click OK.
   d. Record the fit parameters A, B, and C in your data table.

5. Print or sketch your graph.

6. From the definition of half-life, determine the relationship between half-life ($t_{1/2}$, measured in minutes) and decay constant ($\lambda$, measured in $min^{-1}$). Hint: After a time of one half-life has elapsed, the activity of a sample is one-half of the original activity.

7. From the fit parameters, determine the decay constant $\lambda$ and then the half-life $t_{1/2}$. Record the values in your data table.

8. Is your value of $t_{1/2}$ consistent with the accepted value of approximately 2.552 minutes for the half-life of barium-137?

9. What fraction of the initial activity of your barium sample would remain after 25 minutes? Was it a good assumption that the counts in the last five minutes would be due entirely to non-barium sources? How does the curve fit value for B compare to the average count rate between 25 and 30 minutes (determined in step 3)? What does that comparison tell you about the meaning of the curve fit parameter B?

## EXTENSIONS

1. How would a graph of the log of the count rate *vs.* time appear? Using Logger *Pro*, Graphical Analysis, or a spreadsheet, make such a graph. Interpret the slope of the line if the data follow a line. Will correcting for the background count rate affect the shape of your graph?

2. Repeat your experiment several times to estimate an uncertainty to your decay constant measurement.

3. How long would you have to wait until the activity of your barium sample is the same as the average background radiation? You will need to measure the background count rate carefully to answer this question.

# Lifetime Measurement

1. The student pages with complete instructions for data collection using LabQuest App, Logger *Pro* (computers), DataMate (calculators), and DataPro (Palm handhelds) can be found on the CD that accompanies this book. See *Appendix A* for more information.

2. Alert readers may notice that the Preliminary Questions are the same as those in Experiment 24 (Capacitors) of *Physics with Vernier*. This duplication is intentional, as both the decay in capacitor potential in an *RC* circuit and radioactive decay are described by exponential functions. You may wish to call your students' attention to this.

3. Sources are available from a number of suppliers:
   - Spectrum Techniques, (865) 482-9937, Fax: (865) 483-0473, www.spectrumtechniques.com.
   - Flinn Scientific, (800) 452-1261, Fax: (630) 879-6962, www.flinnsci.com.
   - Canberra Industries, (800) 243-3955 Fax: (203) 235-1347, www.canberra.com.

4. Detailed directions for preparing the isogenerator are not given because the method varies with manufacturer. You may want to insert the instructions appropriate to your isogenerator at Step 2 of the Procedure.

5. Students often confuse the decay constant parameter λ with the half-life $t_{1/2}$. The decay constant λ is larger for more rapidly decaying elements and has dimensions of time$^{-1}$, while the half-life has dimensions of time, and is smaller for more rapidly decaying elements. The decay constant λ is equal to the fit parameter C in the Natural Exponential fit of Logger *Pro* and LabQuest. The two parameters can be related in the following manner. After one half-life has elapsed, half of the radioactive nuclei have decayed, and so the activity is also cut in half. From the rate equation we can relate the decay constant to the half life.

$$R = R_0 e^{-\lambda t} \text{ ; at } t = t_{1/2} \text{ we know that } R = \tfrac{1}{2} R_0$$

$$\frac{1}{2} R_0 = R_0 e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}} \text{ . Taking the log of both sides,}$$

$$-\ln 2 = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

   There is sufficient information in the student guide to perform this conversion, although some students with weak algebra skills may have difficulty with it. You may choose to work through this step with your students.

6. The cesium-137 in the isogenerator decays to a metastable state of barium. The metastable barium decays with a half-life of 2.552 minutes by gamma emission, making this system an ideal one for studying in the classroom. A 30 minute experimental run covers almost twelve half-lives, so that the observed activity drops to about 0.3% of the initial value.

7. The lifetime obtained depends strongly on the correct subtraction of background (in this case, non-barium) counts. As written, the activity instructions call for a 30-minute data collection period. If time permits, use a 45 or 60 minute period, and measure the count rate for the final 10 or 15 minutes. A longer experiment will ensure that essentially all the barium will have decayed. The sample data shown here yield a lifetime of 2.50 minutes, but if the background value obtained during the last 10 minutes of a 60 minute run is used, the lifetime changes to 2.57 minutes.

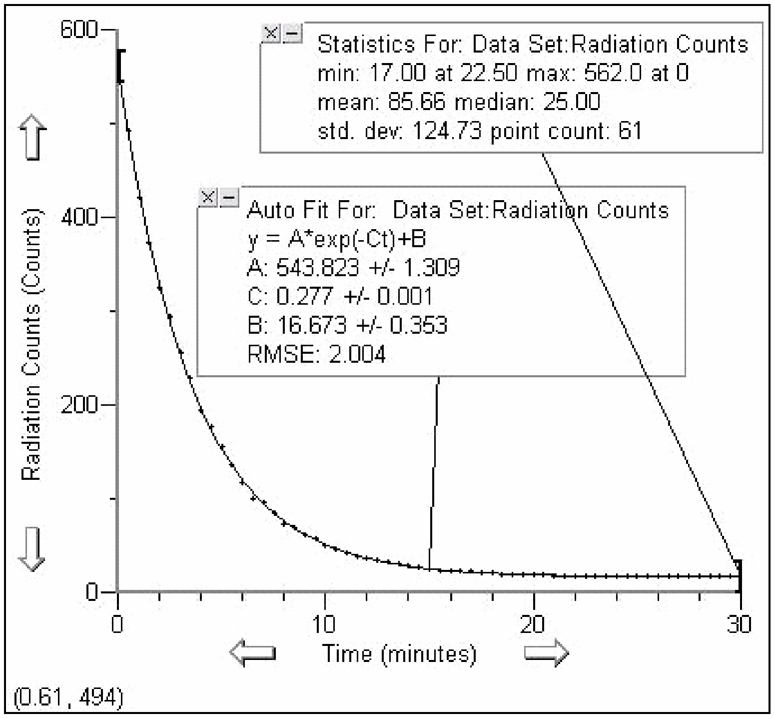
8. Many isogenerators allow some cesium to leak through into the barium extract solution. The cesium results in a nearly constant background activity. This background count is often much larger than the environmental background, and the analysis must take it into account. That is why the experiment is written to run for 30 minutes. The final 5 minutes of data can be used to determine the count rate from the combination of cosmic rays and leaked cesium. If you have an isogenerator that does not leak significant amounts of cesium, you may want to shorten the experiment to fifteen minutes.

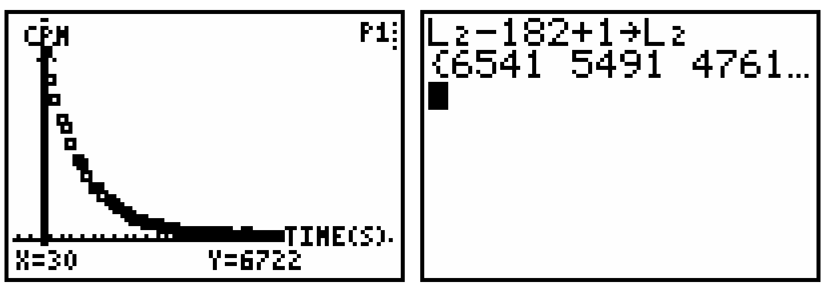
9. In Step 4 of Analysis students perform a curve it on only the first 15 minutes of data. This is important, because the fit will sometimes be poorer if all 30 minutes of data are used. The counts during the first 15 minutes are largely due to the barium, while the counts in the last 15 minutes are mostly from non-barium sources. The many noisy points in the tail of the exponential may unduly influence a fit of the entire run.

   You may want to have students investigate this effect, or to try various selections of data during the first 15 minutes (*e.g.*, 2–13 minutes, or 5–15). The resulting value for the lifetime will vary somewhat, giving an indication of the uncertainty of the measurement. Using our data we get variations about 0.05 minutes around the typical value shown here.

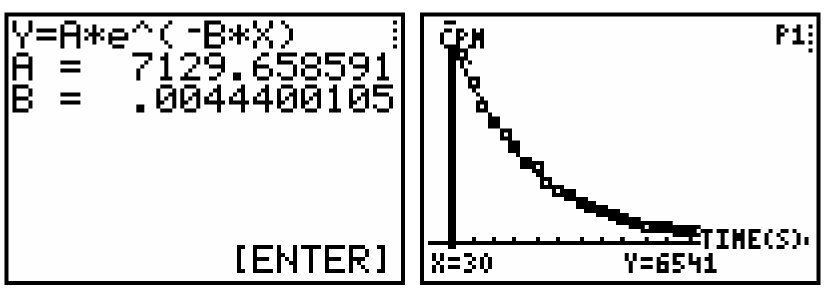
10. Note that the calculator, computer, LabQuest, and Palm versions of the activity use different notation for the fitted equation. Unlike Logger *Pro* and LabQuest, the calculator program DataRad and the Palm program DataPro, uses seconds as the x-axis time unit, so that the exponential fit parameter must be converted from $s^{-1}$ to $min^{-1}$ ($s^{-1} = 60\ min^{-1}$) to obtain a lifetime in $min^{-1}$.

## SAMPLE RESULTS




*Raw data from calculator and background subtraction step.*

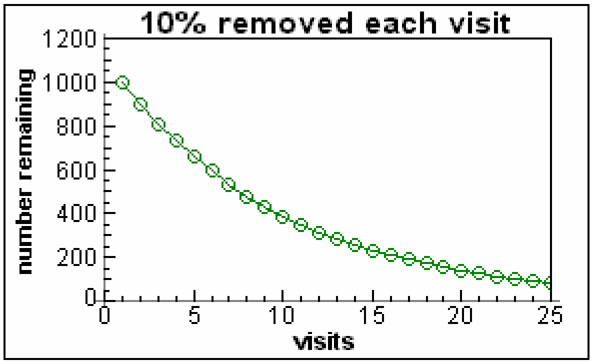


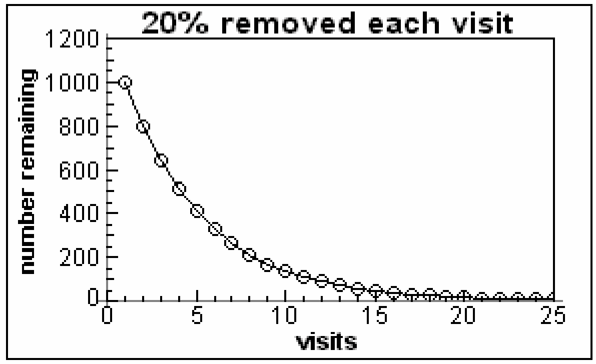

*Exponential fit to the first 15 minutes of data after background subtraction.*

## ANSWERS TO PRELIMINARY QUESTIONS

1. Graph is a decaying exponential. The first few values are 1000, 900, 810… (with integer part of 10% taken each time).

2. Second graph decays more quickly: 1000, 800, 640…




## DATA TABLE

(computer)

| | |
|---|---|
| Average background counts | 17 |
| fit parameters for Y = A exp ( – C*X ) + B | |
| A | 554 |
| B | 17 |
| C | 0.277 |
| $\lambda$ (min$^{-1}$) | 0.277 |
| $t_{1/2}$ (min) | 2.50 |

(calculator)

| fit parameters for Y = A exp ( – B*X ) + C | |
|---|---|
| A | 7129 cpm |
| B | 0.00444 s$^{-1}$ |
| C | 182 cpm |
| $\lambda$ (min$^{-1}$) | 0.266 |
| $t_{1/2}$ (min) | 2.60 |

## ANSWERS TO ANALYSIS QUESTIONS

1. The count rate decreases in time, falling to less than 10% of the initial value. This is consistent with activity being proportional to the amount of remaining radioactive material, since as material decays, less remains, so the activity must decrease.

2. The three graphs have a similar decreasing shape, although the time-axis scale of the barium data is different from that of the candy graphs. The vertical axes have different units (candy remaining and counts/interval). They are similar because in each case the decay process proceeds at a rate proportional to the remaining candies or radioactive nuclei.

6. We start with the rate equation, and then use the definition of the half-life as the time it takes for the activity to drop to one-half the original value:

$$R = R_0 e^{-\lambda t} \text{ ; at } t = t_{1/2} \;\; R = \tfrac{1}{2} R_0$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$-\ln 2 = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

8. The experimental half-life of 2.50 min is close to the accepted value of 2.552 s.

9. After 25 minutes, 0.11% of the original barium activity remains. ($e^{-25*0.272}$ = 0.0011). Most, but not quite all of the original activity has decayed. The assumption that the counts observed during the last five minutes of data collection are due only to non-barium is reasonable. Possibly a better background estimate could be obtained by waiting a longer time.

## ANSWERS TO EXTENSIONS

1. A graph of ln(counts/interval) *vs.* time should be a straight line of negative slope. The slope is $-\lambda$, or the negative of the decay constant. If the background has been subtracted, the graph should be nearly linear. Without background subtraction, the graph will be curved.

2. Results will vary. A collection of lifetime measurements will allow the student to determine a range of values; the extent of that range is a measure of the uncertainty of the measurement. The range of data selected will also influence the measurement, as will the value used for the additive parameter B in the exponential curve fit.

3. Results will depend on the background radiation level. Experiments done at high altitude will experience larger background count rates due to reduced attenuation of cosmic rays by the atmosphere. To measure the background rate, change your set up to count with no source present. Note that the solution obtained from the isogenerator will contain some cesium, raising the count rate further above background from environmental radiation.