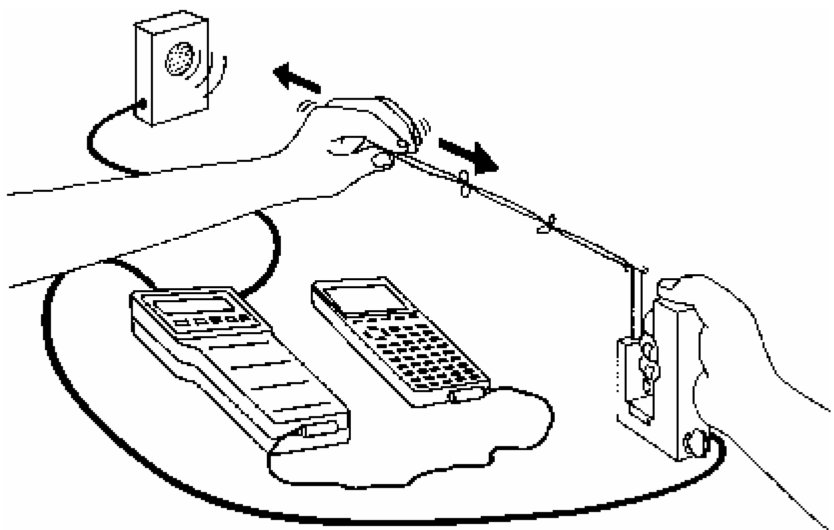
Activity
9

Stretch it to the Limit: The Linear Force Relation for a Rubber Band

When a force is applied to a rubber band, it stretches a certain amount. Exactly how much it stretches depends on the applied force and the characteristics of the rubber band. In general, the more force that is applied, the more it stretches. For rubber bands that are not stretched too much, if you double the force applied, it turns out that the stretch doubles as well. Two quantities, x and y, that change in this way are said to be *proportional*. x and y are related by the constant K in the equation

$$y = Kx$$

In this activity you will use a Force Sensor and a Motion Detector to investigate the relationship between the force applied to a rubber band and the distance that the rubber band stretches. To measure how a rubber band has stretched, we will use the stretched length of the band minus the relaxed length of the band.



OBJECTIVES

- Record force *vs*. stretch data for a rubber band.
- Model force *vs*. stretch data with a proportional relationship.

MATERIALS

CBL 2 or LabPro interface	Motion Detector
TI Graphing Calculator	Vernier Force Sensor
DataMate program	long rubber band (or 3 smaller bands)

PROCEDURE

1. For this activity, you will stretch and relax a rubber band with your hand. A Motion Detector will record the amount the rubber band is stretched while a Force Sensor measures the force of your pull. Be sure not to get closer than 50 cm to the Motion Detector during data

collection. The rubber band being used should be flexible enough to stretch at least 15 cm. Several smaller rubber bands linked together also work well for this activity.

2. Prepare the Force Sensor and Motion Detector for data collection.
   a. Connect the Force Sensor to Channel 1 of the LabPro or CBL 2 interface.
   b. If you are using a Dual Range Force Sensor, set the range switch to 10 N.
   c. Connect the Motion Detector to the DIG/SONIC 1 port of the LabPro or DIG/SONIC of the CBL 2 interface.
   d. Use the link cable to connect the TI Graphing Calculator to the interface. Firmly press in the cable ends.
3. Turn on the calculator and start the DATAMATE program. Press CLEAR to reset the program.
4. Set up the calculator and interface for the Force Sensor.
   a. Select SETUP from the main screen.
   b. If the setup screen lists the Force Sensor, skip to the Step 5.
   c. Press ENTER to select CH 1.
   d. Select FORCE from the SELECT SENSOR menu.
   e. Select the correct force sensor (DUAL R FORCE 10(N) or STUDENT FORCE(N)) from the FORCE menu.
5. To account for the relaxed length of the rubber band, you need to zero both sensors. To do this you have to hold your hands in the position they will be in when data collection begins. You may need to have another person operate the calculator during these steps. Keep in mind that your hand must never get closer than 50 cm to the Motion Detector during data collection, so allow enough room to stretch the rubber band during the activity.
   a. Hold the force sensor with your right hand as shown in the figure, or fix it to a ring stand. The force sensor should not move during data collection. With your left hand hold the rubber band so that it is just barely taut. Your left hand must be directly between the force sensor and the Motion Detector. Keep your hands in this position for now.
   b. Select ZERO from the setup screen.
   c. Select ALL CHANNELS from the select channel screen.
   d. Keep your left hand still, and press ENTER to zero the sensors. This will measure all future distance measurements from the position of your left hand, and all force measurements from the current force, which will be near zero. Don’t move your hand from this position until you begin taking data in the next step.
6. You are now ready to collect force *vs*. stretch distance data.
   a. Select START to begin data collection. Data collection will run for five seconds.
   b. Gently stretch and relax the rubber band, moving your left hand along a line between the Force Sensor and the Motion Detector. Don’t let your hand get any closer than 50 cm from the Motion Detector.
7. After data collection is complete, a menu of available graphs will be displayed. Press ENTER to see a graph of the force as a function of time. Press ENTER to return to the graph selection screen, and press ▼ ENTER to see the distance *vs*. time graph. If you want to repeat data collection, press ENTER, select MAIN SCREEN, and return to Step 6. If you are satisfied with your data, press ENTER and select MAIN SCREEN. Select QUIT to leave DataMate. Follow instructions on your screen to return to your calculator home screen.

8. The graphs shown by DataMate are functions of time. That's not quite what you need, however, since you want force *vs*. distance. To display that graph, you'll need to change your graph settings.

   The distance data reflect the additional stretch of the rubber band. However, because the Motion Detector measures distances as increasing *away* from the detector, you'll need to change the sign of the distance data stored in L6.

   TI-73

   a. Press `(–)` `2nd` [STAT].
   b. Select the list L6 by pressing the digit next to L6 (usually 6).
   c. Press `STO▶` `2nd` [STAT].
   d. Again select the list L6 by pressing the digit next to L6 (usually 6) to complete the expression –L6 → L6.
   e. Press `ENTER` to complete the operation.

   TI-83, TI-83 Plus

   a. Press `(–)` `2nd` [L6] `STO▶` `2nd` [L6] to enter the expression –L6 → L6.
   b. Press `ENTER` to complete the operation.

   TI-86

   a. Press `(–)` L6 `STO▶` L6 to enter the expression –L6 → L6.
   b. Press `ENTER` to complete the operation.

   TI-89, TI-92, and TI-92 Plus

   a. Press `APPS`.
   b. Select 6:Data/Matrix Editor.
   c. Select 1:Current to open the cbldata matrix.
   d. Use the cursor keys to navigate to the c6 column header. c6 should be highlighted.
   e. Notice that c6= is shown in the lower left corner of the screen. Enter –c6 and press `ENTER` to confirm the entry. This entry changes the sign of the contents of the list.
   f. To keep the new values in the c6 column, press `▲` to move the cursor back to the c6 header, and press `ENTER` `CLEAR` `ENTER` to remove the –c6 entry.

9. Now you can plot the force used to stretch the rubber band *vs*. the distance the band was stretched.

   TI-73, TI-83, and TI-83 Plus

   a. Press `2nd` [STAT PLOT] ([PLOT] on the TI-73) and press `ENTER` to select Plot 1.
   b. Use the cursor keys to position the cursor on each of the following Plot1 settings. Press `ENTER` to select any of the settings you change: Plot1 = On, Type = ⸱⸱⸱ , Xlist = L6, Ylist = L2, and Mark = •.
   c. Press `ZOOM` and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.

   TI-86

   a. Press `2nd` [STAT] **PLOT**, and then select **PLOT1**.
   b. Use the cursor keys to position the cursor on On and press `ENTER`.
   c. Press `▼` to position the cursor on the graph type. Select **SCAT**.
   d. Press `▼` to position the cursor on the Xlist Name. Select **L6**.

e. Press [▼] to position the cursor on the Ylist Name. Select **L2**.
f. Press [▼] to position the cursor on the Mark. Select •.
g. Press [GRAPH] and select **ZOOM**; next select **ZDATA** (use [MORE] to reveal **ZDATA**) to draw a graph with the *x* and *y* ranges set to fill the screen with data.

TI-89, TI-92, and TI-92 Plus

a. Press [F2] to select the Plot Setup menu.
b. Using the cursor keys, highlight Plot 1 and press [F1] to select it.
c. If scatter is not displayed, press [►] and select 1:Scatter.
d. Press [▼]. If Box is not displayed, press [►] and select 5:Dot.
e. Press [▼] to move to the x field. In the x field, enter c6, which is the name of your *x* column.
f. Press [▼]. In the y field, enter the name of your *y* column, or c2.
g. Press [ENTER] twice to save the settings.
h. Press [◆] [WINDOW]. Press [F2] to select the Zoom menu.
i. Select 9:ZoomData to fill the graph with your data.

ANALYSIS

1. As mentioned earlier, an approximate model for force and stretch holds that they are proportional. To test this, try plotting the line of proportionality $y = Kx$ with your data. This will take two steps; first, enter your equation.

   TI-73, TI-83, and TI-83 Plus

   a. Press [Y=].
   b. Press [CLEAR] to remove any existing equation.
   c. Enter K*x in the Y1 line. (On the TI-73, access the alphabetic entry screen by pressing [2nd] [TEXT].)
   d. Press [2nd] [QUIT] to return to the home screen.

   TI-86

   a. Press [GRAPH].
   b. Select **y(x) =**.
   c. If equation y1 is not visible, select **INSf** until y1 is displayed.
   d. Press [CLEAR] to clear any y1 equation.
   e. Enter K*x in the y1 line. Use the [x-VAR] key to enter your x.
   f. Press [2nd] [QUIT] to return to the home screen.

   TI-89, TI-92, TI-92 Plus

   a. Press [◆] [Y =].
   b. Select the y1 line using the cursor pad.
   c. Press [CLEAR] to clear any y1 equation.
   d. Enter K*x in the y1 line.
   e. Press [◆] [HOME] ([HOME] on the TI-89) to return to the home screen.

2. Next you will set a value for the parameter *K*, and then look at the resulting graph. To begin with use a slope of one, so $K = 1$. To obtain a good fit, you will need to try several values for

K. Use the steps below to store different values to the parameter *K*. Experiment until you find one that provides a good fit for the data.

TI-73, TI-83, and TI-83 Plus

a. Enter a value for the parameter *K*. Press [STO▶] K [ENTER] to store the value in the variable K.
b. Press [GRAPH] to see your data with the model graph superimposed.
c. Press [2nd] [QUIT] to return to the home screen.

TI-86

a. Enter a value for the parameter *K*. Press [STO▶] K [ENTER] to store the value in the variable K.
b. Press [GRAPH] **GRAPH** to see your data with the model graph superimposed.
c. Press [2nd] [QUIT] to return to the home screen.

TI-89, TI-92, TI-92 Plus

a. Enter a value for the parameter *K*. Press [STO▶] K [ENTER] to store the value in the variable K.
b. Press [◆] [GRAPH] to see your data with the model graph superimposed.
c. Press [◆] [HOME] ([HOME] on the TI-89) to return to the home screen.

Try different values for *K* until you find one that provides a good fit for the data by storing a new value for *K* and then redisplaying your graph. In the space below, record the value of *K* that works best.

K = __________

3. The *K*-value may also be determined algebraically by substituting the *x* and *y* values of one point into the variation equation, $y = Kx$. To choose the *x*- and *y*- coordinates, redisplay your graph and enter trace mode. (TI-73, TI-83, 83 Plus, press [GRAPH] [TRACE]. TI-86, press [GRAPH] and select **GRAPH** then **TRACE**. TI-89 and TI-92, press [◆] [GRAPH] and select [F3] Trace.) Use the cursor keys to move the cursor to a point near the middle of your graph. If your cursor is on your line rather than the data points, press [▲] to shift the focus to the data points. Record the *x* and *y* values, rounded to the nearest hundredth, in the spaces below. Use these values to solve for *K*, and write this value in the table provided.

x	
y	
K	

Are the *K* values determined in Steps 2 and 3 consistent? What might cause them to be slightly different? Use the technique in Step 2 to view the line for this value of *K*.

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4. In addition to your two estimates of the parameter *K*, the calculator can be used to automatically determine the slope and intercept of a straight line. This line can be fit to your data using the steps below.

TI-73

a. Press [2nd] [STAT] and use the cursor keys to highlight CALC.

b. Press the number adjacent to LinReg(ax+b) to copy the command to the home screen.

c. After the LinReg(ax+b) command, press [2nd] [STAT] and select L6 by pressing the number next to L6. Then press [,]. Repeat the procedure to select L2.

d. After selecting L2, press [,] then press [2nd] [VARS].

e. Use the cursor keys to select Y-Vars and press [ENTER] .

f. Press [ENTER] to select Y1 and copy it to the expression.

g. On the home screen, you'll now see the entry LinReg(ax+b) L6, L2, Y1. This command will perform a linear regression with L6 as the *x* and L2 as the *y* values. The resulting regression line will be stored in equation variable Y1. Press [ENTER] to perform the linear regression. Record the regression equation with its parameters on the line below.

h. Press [GRAPH] to see your graph.

TI-83 and TI-83 Plus

a. Press [STAT] and use the cursor keys to highlight CALC.

b. Press the number adjacent to LinReg(ax+b) to copy the command to the home screen.

c. Press [2nd] [L6] [,] [2nd] [L2] [,] to enter the lists containing your data.

d. Press [VARS] and use the cursor keys to highlight Y-VARS.

e. Select Function by pressing [ENTER].

f. Press [ENTER] to copy Y1 to the expression.

g. On the home screen, you'll now see the entry LinReg(ax+b) L6, L2, Y1. This command will perform a linear regression with L6 as the *x* and L2 as the *y* values. The resulting regression line will be stored in equation variable Y1. Press [ENTER] to perform the linear regression. Record the regression equation with its parameters on the line below.

h. Press [GRAPH] to see your graph.

TI-86

a. Press [2nd] [STAT] and select **CALC**.

b. Select **LinR**.

c. Press [2nd] [LIST] and select **NAMES**.

d. Select **L6**, then press [,].

e. Select **L2**, then press [,].

f. Enter y1. To enter a lower-case y, press [2nd] [ALPHA] [Y].

g. On the home screen, you'll now see the entry LinR L6, L2, y1. This command will perform a linear regression with L6 as the *x* and L2 as the *y* values. The resulting regression line will be stored in equation variable y1. Press [ENTER] to perform the linear regression. Record the regression equation with its parameters on the line below.

h. Press [GRAPH] **GRAPH** to see your graph.

TI-89, TI-92, and TI-92 Plus

a. Press [APPS], select 6:Data/Matrix Editor, and then 1:Current to open the data table.

b. Press [F5] to select the Calc menu.

c. Set the calculation type to a linear regression by pressing [►] and then selecting 5:LinReg.

d. Press [▼]. In the x field, enter the name of your *x* column, or c6.

e. Press [▼]. In the y field, enter the name of your *y* column, or c2.

f. Press [▼]. In order to plot the fitted equation, press [►] (and [▲] or [▼] as needed) to highlight the Y1(X) equation variable. Press [ENTER] to select Y1. The regression equation will be saved to this variable.
g. Press [▼]. If YES is displayed for Use Freq and Categories, press [►] and select 1:NO.
h. Press [ENTER] to perform the fit. Record the regression equation with its parameters on the line below.
i. Press [◆] [GRAPH] to see your graph.

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5. How does the slope in the linear regression compare with the *K* values found in Steps 2 and 3? What should be the value of the *y*-intercept from the regression? Explain.

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6. Which equation seems to fit better? Which equation is a better direct variation model? Why?

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7. Explain why the graph is linear even though you stretched the rubber band back and forth in front of the Motion Detector.

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8. How would your data be affected if you used a stiffer, less flexible rubber band? How would this change your *K* value?

EXTENSION

As you were recording motion and force data for the stretched and relaxed rubber band, time values were being recorded simultaneously by the interface. In fact, these data were initially shown to you immediately after data collection. Restart DataMate and inspect your force *vs*. time graph. This plot shows how force values vary with time while you were pulling on the rubber band. Make a prediction about what a plot of stretch distance *vs*. time would look like. Check your prediction by displaying the distance graph. How is the stretch distance *vs*. time plot related to the force *vs*. time plot? Be specific.

Activity

9

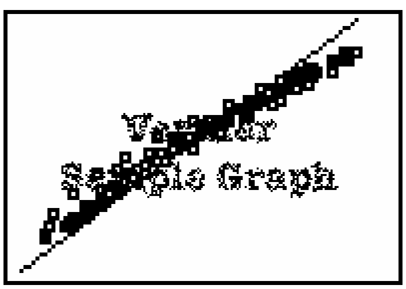
TEACHER INFORMATION

Stretch it to the Limit: The Linear Force Relation for a Rubber Band

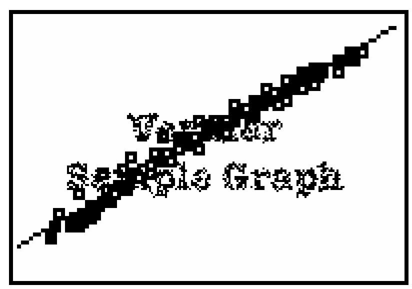
1. A TI CBR can be used in place of a Vernier Motion Detector in this experiment. The CBR must be connected to the interface, and not directly to the calculator.
2. To stretch and relax the rubber band, grasp it with your fingertips or loop it around one finger. The back of your hand should be facing the Motion Detector. During data collection your hand must remain perpendicular to the table surface.
3. The rubber band must remain taut during data collection. If it goes slack the linear relationship between force and distance will not hold.
4. Some Motion Detectors will not be automatically identified by DataMate. If you are using such a detector, you must manually set up DataMate for the detector:

   a. Select SETUP from the main screen.
   b. Press ▼ until the cursor is next to DIG1 (LabPro) or DIG (CBL 2).
   c. Press ENTER to access the SELECT SENSOR menu.
   d. Select MOTION(M).
   e. Select OK to return to the main screen.

SAMPLE RESULTS



Raw Data with proportional model



Data with calculator regression

DATA TABLE

x	xxxx
y	xxxx
K	xxxx

ANSWERS TO ANALYSIS QUESTIONS

Answers have been removed from the online versions of Vernier curriculum material in order to prevent inappropriate student use. Graphs and data tables have also been obscured. Full answers and sample data are available in the print versions of these labs.